

Objective: The student will be able to recognize a sequence, distinguish between arithmetic and geometric sequences, and understand/calculate the parts of each equation.

Sequence – A set of numbers in a specific order. It is discrete, not continuous.

Example 1 - 8, 11, 14, 17, ...

Example 2 - 6, 12, 24, 48 ...

Discrete – Data that can only take certain values.

For example: the number of students in a class (you can't have half a student).

Term – Each number in the sequence.

In Example 1 above – the 2nd term is 11, the 4th term is 17.

Types of Sequences

Formulas:

$$a_n = a_0 + d \cdot n$$

Arithmetic Sequence – A sequence made by adding the same value each time.

$$a_n = \text{value of term you want}$$

$$a_0 = \text{zero term (y-intercept)}$$

$$d = \text{common difference}$$

$$n = \text{\# of term you want to find}$$

Does this sequence and its formula remind you of an equation we've worked with earlier this year?

$$y = mx + b$$

$$d = m$$

$$a_0 = b$$

Example: Write an equation for the following arithmetic sequences.

a. 1, 5, 9, 13, 17, ...

$$a_1 = 1 \quad a_2 = 5 \quad a_3 = 9 \quad a_4 = 13 \quad a_5 = 17$$

$$d = 4$$

$$a_0 = -3$$

$$a_n = -3 + 4n$$

b. -3, -6, -9, -12, ...

$$d = -3$$

$$a_0 = 0$$

$$a_n = 0 - 3n$$

Geometric Sequence – A sequence made by multiplying the same value each time.

Formulas:

$$a_n = a_0 \cdot r^n$$

$$a_n = \text{value of the term you want}$$

$$a_0 = \text{zero term} = \frac{a_1}{r}$$

$$r = \text{common ratio} = \frac{a_2}{a_1}$$

$$n = \text{\# of the term you want to find}$$

Example: Write an equation for the following geometric sequences.

a. 1, 4, 16, 64, 256, ...

$$a_0 = \frac{1}{4} \quad r = 4 \quad \boxed{a_n = \frac{1}{4}(4)^n}$$

b. -32, 16, -8, 4, ...

$$a_0 = 64 \quad r = -\frac{1}{2} \quad \boxed{a_n = 64(-\frac{1}{2})^n}$$

What if you can't easily find the '0' term?

Example: Write an equation for the following arithmetic sequences without using a_0 .

a. 10, 14, 18, 22, ... (use a_1)

$$a_n = 10 + 4(n-1)$$

b. -10, -5, 0, 5, 10, ... (use a_4)

$$a_n = 5 + 5(n-4)$$

Example: Write an equation for the following geometric sequences without using a_0 .

a. 1, 3, 9, 27, 81, ... (use a_5)

$$a_n = 81(3)^{n-5}$$

b. 512, 256, 128, 64, ... (use a_3)

$$a_n = 128\left(\frac{1}{2}\right)^{n-3}$$

What if you're given a term and have to find the 'n'?

Example: Given: $a_n = -30 + 6n$; If $a_n = 48$, what is n ?

$$48 = -30 + 6n$$

$$78 = 6n$$

$$\boxed{n = 13}$$

Example: Given: $a_n = 90 \cdot \left(\frac{1}{3}\right)^n$; If $a_n = \frac{10}{27}$, what is n ?

$$\frac{10}{27} = 90 \left(\frac{1}{3}\right)^n$$

$$\frac{1}{243} = \left(\frac{1}{3}\right)^n$$

$$\boxed{n = 5}$$

Practice:

I. For each sequence, state if it is arithmetic, geometric, or neither. Then, if it's arithmetic or geometric, write a formula for it.

a. 8, 6, 4, 2, 0, -2, ... A

$$a_n = 8 - 2(n-1)$$

b. -4, 12, -36, 108, -324 G

$$a_n = -4(-3)^{n-1}$$

c. 64, 48, 36, ... G

$$a_n = 85\frac{1}{3} \left(\frac{3}{4}\right)^n$$

d. 0, 3, 8, 15, 24 N

e. -24, -26, -18, -10, -2 N

f. 5, 1, $\frac{1}{5}$, $\frac{1}{25}$, $\frac{1}{125}$ G

$$a_n = 25 \left(\frac{1}{5}\right)^n$$

II. Write the first 3 terms of the following sequences.

a. $a_n = -43 + 4n$

$$a_1 = \underline{-39} \quad a_2 = \underline{-35} \quad a_3 = \underline{-31}$$

b. $a_n = 2 \cdot (-3)^n$

$$a_1 = \underline{-6} \quad a_2 = \underline{18} \quad a_3 = \underline{-54}$$

b. $a_n = 3 \cdot (2)^{n-1}$

$$a_1 = \underline{3} \quad a_2 = \underline{6} \quad a_3 = \underline{12}$$

d. $a_n = -163 + 200(n-1)$

$$a_1 = \underline{-163} \quad a_2 = \underline{37} \quad a_3 = \underline{237}$$

Practice.

1. $a_n = 16 + 5n$

a. Find the 80th term.

b. If $a_n = 71$, what is n ?

a) $a_{80} = 16 + 5(80) = \boxed{416}$

b) $71 = 16 + 5n$

$$\boxed{n = 11}$$

2. $a_n = -8 + 2n$

a. Find the 50th term.

b. If $a_n = 44$, what is n ?

a) $a_{50} = -8 + 2(50) = \boxed{92}$

b) $44 = -8 + 2n$

$$52 = 2n$$

$$\boxed{n = 26}$$

3. $a_n = 1080 - 36n$

a. Find the 16th term.

b. If $a_n = 900$, what is n ?

a) $a_{16} = 1080 - 36(16) = \boxed{504}$

b) $900 = 1080 - 36n$

$$\boxed{n = 5}$$

4. $a_n = -600 + 5n$

a. Find the 900th term.

b. If $a_n = -225$, what is n ?

a) $a_{900} = -600 + 5(95) = \boxed{-125}$

b) $-225 = -600 + 5n$

$$375 = 5n \quad \boxed{n = 75}$$

5. $a_n = 3 \cdot 2^n$

a. Find the 10th term.

b. If $a_n = 192$, what is n ?

a) $a_{10} = 3 \cdot 2^{10} = \boxed{3072}$

b) $192 = 3 \cdot 2^n$

$$64 = 2^n$$

$$\boxed{n = 6}$$

6. $a_n = 700(0.8)^n$

a. Find the 20th term.

b. If $a_n = 358.4$, what is n ?

a) $a_{20} = 700(0.8)^{20} = \boxed{8.07}$

b) $358.4 = 700(0.8)^n$

$$.512 = .8^n$$

$$\boxed{n = 3}$$

7. $a_n = 60\left(\frac{1}{2}\right)^n$

a. Find the 10th term.

b. If $a_n = \frac{15}{8}$, what is n ?

a) $a_{10} = 60\left(\frac{1}{2}\right)^{10} = \boxed{.06}$

b) $\frac{15}{8} = 60\left(\frac{1}{2}\right)^n$

$$.03125 = \left(\frac{1}{2}\right)^n$$

$$\boxed{n = 5}$$

8. $a_n = 1000(0.95)^n$

a. Find the 16th term.

b. If $a_n = 902.5$, what is n ?

a) $a_{16} = 1000(0.95)^{16} = \boxed{440.13}$

b) $902.5 = 1000(0.95)^n$

$$.9025 = .95^n$$

$$\boxed{n = 2}$$