

Warm-up: Expand each binomial squared.

1. $(x + 1)^2$

$$\begin{aligned} & (x+1)(x+1) \\ & x^2 + x + x + 1 \\ & \boxed{x^2 + 2x + 1} \end{aligned}$$

2. $(x + 4)^2$

$$\begin{aligned} & (x+4)(x+4) \\ & \boxed{x^2 + 8x + 16} \end{aligned}$$

3. $(x + 7)^2$

$$\boxed{x^2 + 14x + 49}$$

Completing the square

Objective

To write the vertex form of a quadratic equation by completing the square.

Vertex Form

$$y = a(x - h)^2 + k$$

Vertex : (h, k)

Perfect Square Trinomial

A trinomial in the form: $y = x^2 \pm bx + \left(\frac{1}{2}b\right)^2$.
 When factored, it creates two identical factors that can be written as $(x \pm \frac{1}{2}b)(x \pm \frac{1}{2}b)$ OR $(x \pm \frac{1}{2}b)^2$

$$\square = \left(\frac{b}{2}\right)^2$$

Complete the expression to create a perfect square trinomial. Then factor it.

$x^2 + 18x + \boxed{81}$

$$\begin{aligned} b &= 18 \\ \frac{b}{2} &= 9 \\ \left(\frac{b}{2}\right)^2 &= 81 \end{aligned}$$

$$\begin{aligned} & \begin{array}{c} 81 \\ 9 \times 9 \\ 18 \end{array} \\ & (x+9)(x+9) \\ & \boxed{(x+9)^2} \end{aligned}$$

$x^2 - 20x + \boxed{100}$

$$\begin{aligned} b &= -20 \\ \frac{b}{2} &= -10 \\ \left(\frac{b}{2}\right)^2 &= 100 \\ & \boxed{(x-10)^2} \end{aligned}$$

$x^2 + 22x + \boxed{121}$

$$\begin{aligned} b &= 22 \\ \frac{b}{2} &= 11 \\ \left(\frac{b}{2}\right)^2 &= 121 \\ & \boxed{(x+11)^2} \end{aligned}$$

$x^2 + 26x + \boxed{169}$

$$\begin{aligned} b &= 26 \\ \frac{b}{2} &= 13 \\ & \boxed{(x+13)^2} \\ \left(\frac{b}{2}\right)^2 &= 169 \end{aligned}$$

Complete the square to write the quadratic in vertex form

Example: $y = x^2 - 10x - 15$

1. $y + 15 + \square = x^2 - 10x + \square$

2. $y + 15 + \left(\frac{-10}{2}\right)^2 = x^2 - 10x + \left(\frac{-10}{2}\right)^2$

3. $y + 15 + 25 = x^2 - 10x + 25$

4. $y + 40 = (x - 5)^2$

5. $y = (x - 5)^2 - 40$

Notes:

1. Move 'c' to the other side (away from 'a' and 'b')

2. Find the number that will create a Perfect Square

Trinomial and add it to both sides. $\square = \left(\frac{b}{2}\right)^2$

3. Simplify

4. Factor the quadratic (perfect square trinomial)

5. Set equation = y (move constant back)

You Try:

1. $y = x^2 + 22x + 90$

$y - 90 = x^2 + 22x$

$b = 22$

$\frac{b}{2} = 11$

$\left(\frac{b}{2}\right)^2 = 121$

$y - 90 + 121 = x^2 + 22x + 121$

$y + 31 = (x + 11)^2$

$y = (x + 11)^2 - 31$

2. $y = x^2 - 2x - 5$

$b = -2$

$\frac{b}{2} = -1$

$\left(\frac{b}{2}\right)^2 = 1$

$y + 5 = x^2 - 2x$

$y + 5 + 1 = x^2 - 2x + 1$

$y + 6 = (x - 1)^2$

$y = (x - 1)^2 - 6$

3. $y = x^2 + 4x - 9$

$b = 4$

$\frac{b}{2} = 2$

$\left(\frac{b}{2}\right)^2 = 4$

$y + 9 = x^2 + 4x$

$y + 9 + 4 = x^2 + 4x + 4$

$y + 13 = (x + 2)^2$

$y = (x + 2)^2 - 13$

4. $y = x^2 + 16x + 45$

$b = 16$

$\frac{b}{2} = 8$

$\left(\frac{b}{2}\right)^2 = 64$

$y - 45 = x^2 + 16x$

$y - 45 + 64 = x^2 + 16x + 64$

$y + 19 = (x + 8)^2$

$y = (x + 8)^2 - 19$